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The Impact of Overnight Periods on Option Pricing

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Abstract

This paper investigates the effect of closed overnight exchanges on option prices. During the trading day asset prices follow the literature's standard affine model which allows asset prices to exhibit stochastic volatility and random jumps. Independently, the overnight asset price process is modelled by a single jump. We find that the overnight component reduces the variation in the random jump process significantly. However, neither the random jumps nor the overnight jumps alone are able to empirically describe all features of asset prices. We conclude that both random jumps during the day and overnight jumps are important in explaining option prices, where the latter account for about one quarter of total jump risk.

KEYWORDS: Derivative pricing, Jump diffusion, Stochastic volatility.

JEL CLASSIFICATION: G11, G13

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1 Introduction

As a result of the shortcomings in the classical Black-Scholes model for option pricing, two streams of literature can be identified. The first stream extends the Black-Scholes framework to time varying volatility and the occurrence of random jumps in the underlying stock price process. Hull and White (1987) derive option prices in a stochastic volatility model under the assumption that volatility risk is idiosyncratic. Heston (1993) gives closed form option pricing formulas using a mean reverting volatility process and an explicit volatility risk premium. Parallel to this, Merton (1976) motivates that the occurrence of abnormal events can be modelled by a jump component in the underlying stock price process. That paper discusses the implications for option pricing in case jumps are modelled as a compound Poisson process and under the assumption that jump risk is not priced in the market. The models derived in Heston (1993) and Merton (1976) can be merged in the affine jump diffusion framework of Duffie, Pan, and Singleton (2000), where asset returns and variances are driven by a finite number of state variables. The second stream of literature uses more general Lévy processes instead of Brownian Motion and the compound Poisson process as driving factors for asset returns. If the parsimonious variance gamma process is assumed to be the stochastic process for underlying stock returns, Madan, Carr, and Chang (1998) derive closed form expressions for the density of asset returns and option prices. Stochastic volatility models driven by Lévy processes are studied in Carr, Geman, Madan, and Yor (2003), among others.

From the empirical results concerning the aforementioned models, it is evident that jumps are important in explaining characteristics of asset returns and option prices, see, for example, Bakshi, Cao, and Chen (1997), Pan (2002), Andersen, Benzoni, and Lund (2002), and Madan, Carr, and Chang (1998). Using a parametrically specified pricing kernel, Pan (2002) provides evidence that both volatility risk and jump risk are priced in the SPX options market. Coval and Shumway (2001) end up with a similar conclusion using returns on option positions. These positions are constructed such that the value of the positions is only sensitive to changes in the two risk factors. The Lévy literature also provides support for priced volatility and jump risk since the parameter estimates under the objective and the risk neutral measure are generally significantly different. For instance, Madan, Carr, and Chang (1998) find significant negative skewness under the risk neutral probability measure while this is not present in their objective parameter estimates. The differences between the objective and the risk neutral distributions are indicative of the presence of a price for crash risk in options markets. However, it is not always obvious how to infer market prices of risk from the estimation results, because a parametric pricing kernel that defines risk prices, is usually not specified in this literature. On the whole, it is clear from both streams of literature that jumps, next to stochastic volatility, are important in explaining observed patterns in asset returns and option prices.

The present paper considers the jump process in more detail by focusing on jumps in

asset prices that are inherent to overnight market closure. Most of the empirical research cited above, uses daily returns. These returns are calculated using the last tick price on the exchange of each trading day. However, the exchange is closed a large part of the day and information that arrives during the closing time cannot be immediately incorporated in stock prices. For instance, European investors use information revealed in US stock markets, by submitting orders to their exchange before the opening. This means that the opening price of the exchange reflects overnight information. While the effect of market closure on stock prices has been considered in, for example, French and Roll (1986), we are not aware of any paper that focuses on the implications for option pricing. In this paper we stress the difference in information by using different processes driving intraday and overnight returns, respectively. In particular, in the spirit of Andersen, Benzoni, and Lund (2002), we assume, a continuous part with stochastic volatility (reflecting the normal vibrations in the stock price) and a jump part (modelling the arrival of important new information) during the day. Furthermore, the “normal” overnight change in the stock price is modelled by means of a single jump. Additional random jumps due to important news releases are not excluded in the overnight period. We investigate the theoretical and empirical implications of this added factor on option prices.

We find, for the SPX market over two separate periods, that both random jumps and overnight jumps are important for option pricing. In particular, the overnight jump component accounts for approximately one quarter of total jump variation. Moreover, the inclusion of overnight jumps leads to different parameter estimates for the stochastic volatility and random jump part of the stock price process. This will have important consequences for hedging these risks.

The organization of the paper is as follows. Section 2 provides the theoretical formulation and motivation of the model under both the objective and risk neutral measure. We also give a closed-form option pricing formula in the spirit of Heston (1993). Section 3 describes the data and discusses the estimation procedure. In Section 4 the empirical results are presented. Section 5 concludes. Mathematical details are gathered in the Appendix.

2 The Overnight Jump Model

2.1 Stock price process

Financial markets all over the world do not allow for continuously trading stocks, interest rates products, and derivatives. Trading usually starts in the morning hours local time and ends in the late afternoon or in the evening (Germany). Of course, it is possible for individual and institutional investors to do 24 hours trading all over the world: by the time London closes, Wall Street is already open and when the US markets stop trading, Asian exchanges have already opened their doors. Due to increasing globalization and financial market integration,

economies and firms from various countries are interrelated. As a consequence, changes in the value of financial instruments on different exchanges are not independent. This does not only hold if exchanges are open simultaneously, but also if one market is closed. In case an exchange is closed, relevant news cannot be immediately incorporated in prices. For instance, a high closing of stocks traded on the Dow Jones usually has a positive effect on stock price openings in Europe. All news that is important for the value of a particular stock should ideally be processed in the opening price of the stock. The difference between the closing price and the opening price the next day can be seen as a measure of the revealed information all over the world during the overnight period.

Up to now, the overnight period in financial markets has not been considered in the derivative pricing literature. This paper tries to fill this gap by explicitly modelling this period through an additional jump process. The jump in the stock price process exactly models the observed overnight return. Of course, closed markets also imply that we have to add an overnight jump to the money market process. However, as the interest rate sensitivity of stock derivatives is usually found to be rather low, the implications of this will be rather limited.

The money market process is given by, assuming a possibly different (annualized) risk-free interest rate r during the trading day and r^o during the overnight period,

$$\frac{dB_t}{B_{t-}} = rdt + d \sum_{i=1}^{\lfloor 252t \rfloor} \left\{ \exp \left(\frac{r^o}{252} \right) - 1 \right\}, \quad (2.1)$$

i.e., $B_t = \exp \{rt + r^o \lfloor 252t \rfloor / 252\}$, where $\lfloor \cdot \rfloor$ denotes the floor function.

In this paper we use the equivalent martingale method for pricing options. In comparison to the standard Black and Scholes (1973) framework, there are some added risk factors that make the market incomplete with respect to the traded financial securities. A consequence is the non-uniqueness of the equivalent martingale measure \mathbb{Q} . Motivated by, for example, the Breeden (1979) consumption based model, the value process of the underlying in transaction time under the real neutral probability measure \mathbb{Q} is defined by,

$$\begin{aligned} \frac{dS_t}{S_{t-}} &= rdt + \sigma_t dW_t^S + d \sum_{i=1}^{N_t} (Y_i - 1) - dA_t + d \sum_{i=1}^{\lfloor 252t \rfloor} (V_i - 1), \\ \log Y_i &\sim N \left(\log(1 + \mu_{RJ}) - \frac{1}{2} \sigma_{RJ}^2, \sigma_{RJ}^2 \right), \\ \log V_i &\sim N \left(\sum_{i=1}^{\lfloor 252t \rfloor} \frac{r^o}{252} - \frac{1}{2} \frac{\sigma_{OJ}^2}{252}, \frac{\sigma_{OJ}^2}{252} \right), \\ \text{Corr}_t(dW_t^V, dW_t^S) &= \rho dt, \end{aligned} \quad (2.2)$$

where $\{W_t^S\}$ is a standard Brownian Motion independent of the Poisson process $\{N_t\}$ with,

$$N_t \sim \text{Pois} \left((1 - c) \lambda t + c \lambda \frac{\lfloor 252t \rfloor}{252} \right).$$

Both $\{W_t^S\}$ and $\{N_t\}$ are also assumed to be independent of sequences of jumps $\{Y_i\}$ and $\{V_i\}$. Note that the volatility model with jumps of Bakshi, Cao, and Chen (1997) and Andersen, Benzoni, and Lund (2002) is obtained by setting the parameter c equal to zero and by deleting the last sum covering the overnight jump part in (2.2). The time-varying volatility process $\{\sigma_t^2\}$ will be defined below.

Note that the random jump distribution of the Y 's is parametrized such that a single jump multiplies, in expectation, the price by $1 + \mu_{RJ}$. On a yearly basis, due to the random number of jumps, this implies an expected instantaneous drift term $\{A_t\}$, see the Appendix, that needs to be compensated in (2.2) to keep the martingale property of the discounted price process.

Our contribution consists of an extra jump term that is added to the stock price process. For simplicity we count weekends as a single night and we have 252 days a year. At each time which is a multiple of $1/252$, an overnight period is inserted. Each overnight period results in a stock return that is reflected by the jump V_i . Note that the random jump process (interpreted before as, for example, news releases) will also be active during the overnight periods but possibly at a different rate. The parameter c allows the random jumps to have a different intensity during the trading day compared to the overnight period. The expected number of *random* Y -jumps during one calendar year (in addition to the 252 V -jumps) is equal to λ . Finally, note, as required, that the \mathbb{Q} -expected yearly return on the stock price in our model is given by,

$$E_t S_{t+1}/S_t = \exp\{r + r^o\}.$$

The specification of the stochastic variance process in (2.2) is taken from Heston (1993),

$$\begin{aligned} d\sigma_t^2 &= -\kappa(\sigma_t^2 - \sigma^2) dt + \sigma_\sigma \sigma_t dW_t^V. \\ \text{Corr}_t(dW_t^V, dW_t^S) &= \rho dt, \end{aligned} \tag{2.3}$$

where κ is the speed of mean reversion, σ^2 is the long run mean of the variance, and σ_σ the volatility of volatility. This specification allows a negative premium for volatility risk, see, for example, Bakshi and Kapadia (2003) for theoretical and empirical evidence. It has been often observed that a large decline in the stock price is accompanied by a positive shock in volatility levels. This is captured by means of the parameter ρ .

2.2 Option pricing

Given the risk neutral processes in (2.2), a standard plain vanilla call option can be priced using,

$$C_t = B_t E_t \left(\frac{\max(S_T - X, 0)}{B_T} \right),$$

	January 1992–August 1997			July 1999–November 2003		
	close-close	open-close	close-open	close-close	open-close	close-open
average	13.2%	5.5%	7.7%	-4.3%	-3.5%	-0.8%
std.dev	10.5%	9.9%	2.7%	20.6%	18.9%	7.9%
skewness	-0.28	-0.26	-2.54	0.13	0.21	0.25
kurtosis	4.8	4.7	40.8	4.6	5.9	10.4

Table 1: Summary statistics S&P-500 returns during the low volatility period January 1, 1992–August 27, 1997, and the high volatility period July 9, 1999–November 27, 2003.

where T is the maturity and X is the strike price of the option. Following Heston (1993), we show in Appendix A that the pricing formulas for the value of a call option C and a put option P at time t can be simplified as,

$$C_t = S_t P_1 - X e^{-r(T-t)-nr^o/252} P_2, \quad (2.4)$$

$$P_t = X e^{-r(T-t)-nr^o/252} (1 - P_2) - S_t (1 - P_1). \quad (2.5)$$

where the probabilities P_1 and P_2 are given by (A.1) and (A.2), and $n = \lfloor 252T \rfloor - \lfloor 252t \rfloor$ denotes the remaining number of overnight periods till maturity. The proof uses the independence of the overnight process and the intraday process and the fact that the trading day part of the model is an affine jump diffusion in the spirit of Duffie, Pan, and Singleton (2000).

3 Data and Estimation Issues

In the previous section we motivated that different processes describe the intraday and overnight returns. We focus on the S&P-500 index in two periods: a low volatility period from January 1, 1992 until August 27, 1997 and a high volatility period from July 9, 1999 until November 27, 2003.

To assess the effects of market closure in an intuitive informal way, Table 1 shows the sample statistics of the close to close, open to close, and close to open returns series for the respective sample periods. From the standard deviations in Table 1 it is clear that the overnight return is an important part of the total daily return in both the first and the second period. As the sample standard deviation of the overnight returns is lower than the standard deviation of the intraday returns, one may conclude that information important for S&P stocks generally arrives during trading hours. Information of significant importance during the night often leads to a high, either positive or negative, return on the S&P-500 explaining the high kurtosis values of overnight returns in Table 1.

Finally, we have the daily closing option quotes of SPX options for both sample periods.

		January 1992–August 1997				July 1999–November 2003			
CALLS		days to expiration			subtotal	days to expiration			subtotal
	moneyiness	<60	60-180	>180		<60	60-180	>180	
ITM	< 0.97	0.210	0.171	0.140	36376	0.319	0.277	0.252	26418
		14753	14802	6821		12618	10918	2882	
ATM	0.97-1.03	0.136	0.138	0.152	33875	0.222	0.225	0.238	17685
		14611	13693	5571		7945	6834	2906	
OTM	> 1.03	0.124	0.118	0.172	19984	0.302	0.234	0.208	29826
		4768	9380	5836		13886	13742	2198	
subtotal		34132	37875	18228	90235	34449	31494	7986	73929
PUTS		days to expiration			subtotal	days to expiration			subtotal
	moneyiness	<60	60-180	>180		<60	60-180	>180	
OTM	< 0.97	0.191	0.173	0.172	34706	0.330	0.283	0.249	29418
		12912	14729	7065		14321	12132	2965	
ATM	0.97-1.03	0.137	0.139	0.151	34170	0.220	0.222	0.233	17653
		14690	13771	5709		7942	6805	2906	
ITM	> 1.03	0.163	0.125	0.130	25894	0.256	0.220	0.205	23014
		8513	11259	6122		10223	10630	2161	
subtotal		36115	39759	18896	94770	32486	29567	8032	70085

Table 2: Summary statistics on SPX call and put option implied volatilities. The reported numbers are implied volatilities of options on the S&P-500 index corresponding to the average last tick before 3:00 PM and the total number of observations for each maturity category. The sample periods are January 1, 1992, to August 27, 1997, and July 9, 1999, to November 27, 2003, respectively.

These data are extracted from the ABN-Amro Asset Management option database. Following Bakshi, Cao, and Chen (1997), for each day in the sample, only the midprice based on the last reported bid-ask quote (prior to 3:00 PM Central Standard Time) of each option contract is used for estimation. Of course, the aforementioned S&P-500 index levels are measured at the same time. Following Jackwerth and Rubinstein (1996), we assume that the dividend amount and timing expected by the market is identical to the dividends actually paid on the index. We use interpolated LIBOR rates as a proxy of the intraday risk-free rate. In addition, we use information on overnight interest rates in the US market from Bloomberg.

Table 2 provides descriptive statistics on call and put option prices (stated in terms of Black-Scholes implied volatilities) that

1. have time-to-expiration of greater than or equal to six calendar days,
2. have a bid price of greater than or equal to 3/8\$,
3. have a bid-ask spread of less than or equal to 1\$,
4. have a Black-Scholes implied volatility greater than zero and less than or equal to 0.70, and satisfy the arbitrage restriction,

$$C(t, T) \geq \max \left(0, S_t e^{-q(T-t)} - X e^{-r(T-t)} \right),$$

for call options and a similar restriction for put options. In this formula X is the option exercise price, q the dividend rate, and r the continuously compounded intraday risk-free rate.

From the numbers in Table 2, well known patterns in implied volatilities across strikes and maturities are recognized. The volatility skew or smile is clearly present for all option categories but one. This exceptional category is probably less frequently traded. From the return data in Table 1, it is clear that the 1992–1997 sample period can be characterized as a low volatility period and the 1999–2003 sample as a high volatility period. This characterization of both periods also becomes clear from the implied volatilities in Table 2, since they are consistently on a higher level across strike prices and maturities in the 1999–2003 sample period. Christensen and Prabhala (1998), among others, provide evidence for a high correlation between realized volatility and Black-Scholes implied volatility.

In this paper we extract information about \mathbb{Q} -parameters from the option prices since our focus is on the influence of overnight jumps on these options. The practical implementation of the estimation procedure is straightforward and follows Bakshi, Cao, and Chen (1997). For a particular day t , a set of N options is chosen for which the closing price is observed. Henceforth, the i -th option price in this set will be denoted by O_{it}^{obs} . For all options, related values as strike price, remaining time to maturity, risk-free interest rates, and (dividend discounted) value of the underlying are observed as well. Subsequently, we have a model price of option i at time t , say O_{it}^{model} , that is a function of the structural \mathbb{Q} parameter vector $\theta = (\mu_J, \sigma_{RJ}, \lambda, \sigma_{OJ}, \kappa, \sigma, \sigma_\sigma, \rho)$ and the unobservable instantaneous variance σ_t^2 . For a particular time t the estimated parameter vector is determined from,

$$\left[\hat{\theta}_t, \hat{\sigma}_t^2 \right] = \underset{\theta, \sigma_t^2}{\operatorname{argmin}} \sum_{i=1}^N \left(\frac{O_{it}^{\text{model}} - O_{it}^{\text{obs}}}{O_{it}^{\text{obs}}} \right)^2. \quad (3.1)$$

This objective function implies that we focus on fitting the steepness of the observed (Black-Scholes) implied volatility skews or otherwise stated the tails of the market implied risk neutral distribution, see Britten-Jones and Neuberger (2000). The procedure is repeated for each day

in both samples resulting in two time series of estimators. Similar procedures are applied to option pricing models in Bakshi, Cao, and Chen (1997) and Madan, Carr, and Chang (1998). In the implementation of the procedure above we only use the out-of-the money options (for low strikes put options and for high strikes call options), since these options are generally more liquid than in-the-money options.

4 Empirical Results

This section gives the estimation results that are obtained by applying the data and estimation techniques as described in Section 3 to the model formulated in Section 2. First, as a benchmark, we present results for the standard stochastic volatility model (SV) and the stochastic volatility model with random jumps (SVRJ). These results are followed by a discussion of the results in the extended model including overnight jumps. These results are presented both in a setting with only stochastic volatility during the day (SVOJ) as well as in a setting where also random jumps are possible (SVOJRJ).

4.1 Standard option pricing models

In this subsection we present the results for the SV-model and the SVRJ-model to make our results comparable to those of Bakshi, Cao, and Chen (1997). Their model specification and their estimation techniques are similar to the ones we employ. For both sample periods described in Section 3, Table 3 gives an overview of the estimation results of the risk neutral parameters.

For the SV-model, Table 3 confirms that the average instantaneous volatility in the 1992–1997 sample is low in comparison to, for example, the estimated values in Bakshi, Cao, and Chen (1997) over the period June 1988 to May 1991. In the 1999–2003 sample the average instantaneous volatility is higher. In comparison to Bakshi, Cao, and Chen (1997), we also observe different parameter estimates of σ_σ , κ , and σ . One obvious explanation for these differences is the different sample periods used. Furthermore, Bakshi, Cao, and Chen (1997) focus on absolute pricing errors while in this paper relative pricing errors are considered, see (3.1). By using relative pricing errors the misspecification of the SV-model becomes more apparent since a high value of σ_σ is necessary to fit empirically observed implied volatility curves.

To address this issue in more detail, we consider the usual situation where the option implied volatility curve for short term options is downward sloping in the strike price. The steepness of the implied volatility curve provides information about the risk neutral distribution of the underlying index at the maturity date. The steeper the implied volatility curve for a certain strike price region, the more probability mass in that particular region of the

	January 1992–August 1997				July 1999–November 2003			
	SV	SVRJ	SVOJ	SVRJOJ	SV	SVRJ	SVOJ	SVRJOJ
μ_{RJ}		-6.3%		-7.2%		-11.5%		-8.5%
		(3.9%)		(3.6%)		(7.6%)		(5.8%)
σ_{RJ}		8.8%		6.7%		13.8%		11.4%
		(4.2%)		(2.7%)		(10.0%)		(6.8%)
λ		0.60		0.54		0.63		1.18
		(0.05)		(0.42)		(0.11)		(1.05)
σ_{OJ}			7.5%	5.1%			6.9%	7.7%
			(2.8%)	(2.6%)			(5.2%)	(4.6%)
κ	1.67	3.55	1.62	3.32	1.60	3.90	1.62	3.27
	(0.96)	(1.00)	(0.01)	(1.08)	(0.51)	(0.11)	(0.02)	(3.55)
σ	16.0%	11.6%	16.0%	11.2%	15.9%	11.3%	16.0%	11.9%
	(4.3%)	(3.5%)	(1.0%)	(4.8%)	(1.5%)	(3.1%)	(1.7%)	(5.2%)
σ_σ	61.1%	40.0%	92.4%	50.6%	86.8%	39.3%	79.2%	56.7%
	(18.7%)	(0.3%)	(32.2%)	(28.9%)	(18.7%)	(11.2%)	(38.9%)	(40.3%)
ρ	-0.69	-0.59	-0.88	-0.71	-0.64	-0.53	-0.89	-0.70
	(0.15)	(0.20)	(0.13)	(0.23)	(0.18)	(0.07)	(0.15)	(0.22)
σ_t	14.4%	11.7%	11.9%	9.5%	24.8%	20.2%	22.3%	17.4%
	(3.0%)	(3.2%)	(3.4%)	(3.4%)	(5.3%)	(5.0%)	(7.0%)	(7.4%)
SSE	0.70	0.16	0.40	0.12	0.70	0.22	0.49	0.10
	(0.58)	(0.15)	(0.35)	(0.10)	(0.66)	(0.31)	(0.53)	(0.13)

Table 3: Implied average parameter estimates in SV, SVRJ, SVOJ, SVRJOJ models using option data on the S&P-500 from the low volatility period January 1, 1992, until August 27, 1997, and the high volatility period July 9, 1999, until November 27, 2003. Standard deviations of the daily parameter estimates in brackets.

implied risk neutral distribution.

There is an enormous literature on methodologies that extract information about the risk neutral distribution from option prices, see for example Britten-Jones and Neuberger (2000). Because we minimize squared relative errors, the fit of cheaper options (short term OTM puts and calls) is relatively more important compared to the more expensive options in the sample (long term ATM and ITM puts and calls). Stated differently, we focus more on the tails of the market implied risk neutral distribution. The negative slope of the implied volatility curve for short term options forces the optimization algorithm to choose parameter values that are able to generate negative skewness in the risk neutral distribution. The desired skewness can be obtained both from ρ and σ_σ . In more detail, the SV-estimates would imply a volatility of volatility $\sigma_\sigma\sigma_t$ of 0.09 in the low volatility period and a volatility of volatility of 0.21 in the high volatility period while in empirics volatility of volatility is estimated around 0.05 in low volatility markets and 0.12 in high volatility markets.

The estimation results show that part of the misspecification in the SV-model is solved by adding random jumps to the option's underlying value. Compared to the SV-estimates, the parameter estimates of σ_σ and ρ are much smaller in the SVRJ-model which is due to the appearance of (on average) negative jumps that capture (part of) the negative skewness in the implied risk neutral distribution. The three parameter random jump size process combined with stochastic volatility is superior to the SV-model in describing the tails of the market implied risk neutral distribution and fitting the option data.

Comparing the results for both sample periods, we see that the instantaneous volatility in the SVRJ-models is lower on average than the SV-model. This makes sense, since the total variation in the underlying value is now divided in the variation of a jump component and the variation that stems from the stochastic volatility part of the model. The variance in the log-return due to the jumps is given by,

$$\text{Var}_t \left(\sum_{i=1}^{N_{t+1}-N_t} \log Y_i \right) = \lambda \sigma_{RJ}^2 + \lambda \left(\log(1 + \mu_{RJ}) - \frac{1}{2} \sigma_{RJ}^2 \right)^2,$$

and hence the standard deviation due to the random jump part is given by 8.7% and 15.4% in the respective sample periods. Taking σ_t as a proxy of the standard deviation of the continuous part of the underlying value process, we obtain that approximately one third of the total variance is due to random jumps. Moreover, if the variance of the random jump part is added to the estimate of σ_t^2 , then, for both samples, the total variance in the SV-model is almost identical to the total variance in the SVRJ-model.

Summarizing, we see that the parameter estimates in the SVRJ-model are much more in line with the findings of Bakshi, Cao, and Chen (1997) than in the SV-model case. The addition of the random jump component stabilizes the stochastic volatility parameters to more reasonable levels and, hence, reduces the misspecification of the model.

4.2 Option pricing models with overnight jumps

As the goal of the present paper is to assess the importance of overnight trading halts for derivative pricing, we compare the estimation results for SVOJ- and the SVRJOJ-models with the results in the previous subsection.

As a first remark, note that the yearly log-return on a risk-free investment of one dollar in the model with overnight jumps is equal to $r + r^o$. Since trading takes place approximately 6.5 hours a day, we divide the annualized risk-free rate during trading periods and the annualized overnight risk-free rate in the proportions $\frac{1}{4}$ and $\frac{3}{4}$, respectively. Secondly, we observe that the parameter c is not present in the option pricing formulas (2.4)–(2.5), i.e., the different risk neutral jump intensities during trading periods and overnight periods cannot be identified from option data, only λ , the total expected number of random jumps during a calendar year, is determined by option prices.

Table 3 shows that the parameter estimates in the SVOJ-model are quite similar to the ones resulting from the SV-model. Again, just as discussed for the SV-model, the parameters σ_σ and ρ are too extreme in the SVOJ-model. This leads to the conclusion that the inclusion of overnight jumps only, fails to produce the desired skew in the risk neutral distribution. Moreover, as we have seen in the SVRJ-model, the attributed proportion of the total variance due to jumps is approximately one third. Especially in the second sample period, the SVOJ-model fails to reproduce this result. Taking σ_t as a proxy of the standard deviation of the continuous part, the total variance is given by $\sigma_t^2 + \sigma_{OJ}^2$. We obtain that the jump proportion of the variance is slightly less than one third (28%) in the first period but that it is far too low (9%) in the second period. Since jumps play a more dominant role in high volatility periods, this once more indicates that the SVOJ-model does not meet our purposes. A final objection against the SVOJ-model is the fit to the option data. Of course, the SVOJ-model beats the classical SV-model but the increased fit due to overnight jumps, although not negligible, is low in comparison to the inclusion of random jumps as in the SVRJ-model. We conclude that replacement of the random jumps in the SVRJ-model by a single overnight jump is not sufficient. However, the question whether overnight jumps influence option prices, remains open. This issue will be tackled in the next paragraph.

The estimation results for the SVOJRJ-model clearly outperform the models discussed before. We see, that in comparison to the SV-, SVRJ-, and the SVOJ-models, the SVRJOJ-model improves the fit of option prices considerably. The addition of random jumps to the SVOJ-model has the same effect on parameters σ_σ and ρ as the addition of random jumps to the SV-model. The reasoning is also the same: the random jump part captures (part of) the negative skewness in the risk neutral distribution required to fit option prices that otherwise could only partly be captured by large changes in the parameters σ_σ and ρ . Comparing the remaining parameters in the SVRJOJ-model with the SVRJ-model leads to some first

obvious conclusions. Since overnight jumps are included, the parameter estimates of the random jump distribution are less dominant and since the total variance has to be divided over three terms, the estimated variance of the continuous part diminishes. One striking difference is the change in the estimated intensity λ . In the first sample period, the estimated value decreases as expected since additional jumps are added. However, in the high volatility period, the intensity is almost doubled compared to the SVRJ-model. This effect is greatly offset by the much lower value of σ_{RJ} . Probably, in high volatility periods, the model fits much more smaller jumps and due to the effect of the overnight jump, the SVRJOJ-model is better able to identify the smaller jump intensity.

In the same spirit as in the previous subsection, the total variance of the log-returns can be split into three parts: a first component arrives from the stochastic volatility term σ_t , and the two remaining components stem from both the random jumps and the overnight jumps. The trading period's variance consists of the variance of the continuous component (stochastic volatility) and (part of) the random jump component. The non-trading overnight period variance is due to the remaining part of the random jump component and the overnight jumps. Similar to the continuous trading model without overnight periods, we derive the variance in the log-return due to the jumps in the extended models SVOJ and SVRJOJ,

$$\text{Var}_t \left(\sum_{i=1}^{N_{t+1}-N_t} \log Y_i \right) = \lambda \sigma_{RJ}^2 + \lambda \left(\log(1 + \mu_{RJ}) - \frac{1}{2} \sigma_{RJ}^2 \right)^2 + \sigma_{OJ}^2.$$

Given the estimates of the SVRJOJ-model in Table 3, we obtain estimated standard deviations due to the jumps of 9.1% and 17.9%, in the respective periods. These values can be split into a standard deviation of 7.5% (16.1%) due to the random jumps and 5.1% (7.7%) due to the overnight jumps in the first (second) sample period. The proportion of the total variance due to jumps has increased to around 50% in both sample periods. On average 25% of this part has to be attributed to the overnight jumps, once more indicating that the inclusion of overnight jumps is nonnegligible.

This section showed that the most appealing model is clearly the SVRJOJ-model, allowing for difference in intraday asset return variance and overnight asset return variance. The SVRJOJ-model fits empirical option prices best in two different sample periods. Since this model contains the overnight jump part, which covers approximately one quarter of total jump variance, we found evidence that overnight periods are important and have a significant impact on option prices.

5 Conclusion

We presented an option pricing model that explicitly models the influence of non-trading overnight periods on option prices. We conclude that both random jumps during trading

periods and the overnight jump are important in explaining observed option prices. We found that in two sample periods, of which the first can be characterized as a period of low volatility and the second as a period of high volatility, the added jump component covers a significant amount of the variation in the underlying value (risk neutral) process. In more detail, we found that the overnight jump part covers approximately one quarter of total jump variation. Moreover, fifty percent of the daily variance is explained by jumps, either random or overnight. Furthermore, the empirical results reveal that model including the overnight jump component gives a better fit of empirical option prices than the traditional asset pricing models. Finally, we showed that a model containing only overnight jumps in combination with stochastic volatility has the same problem as a pure stochastic volatility model: the estimated volatility of volatility is too large in comparison to the volatility of volatility extracted from volatility series. Hence, this paper concludes that total jump risk should be separated in random jump risk during the trading day and overnight jump risk.

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A Option Pricing Formula

We derive the theoretical formula for a plain vanilla call option given the risk neutral process in (2.2). The put price follows similarly. Using Ito’s Lemma, the stochastic differential of $\log S_t$ is,

$$d \log S_t = \left(r - \frac{1}{2} \sigma_t^2 \right) dt + \sigma_t dW_t^S + d \left(\sum_{i=1}^{N_t} \log Y_i \right) - dA_t + d \left(\sum_{i=1}^{\lfloor 252t \rfloor} \log V_i \right).$$

Following Scott (1997), the call option value formula is given by,

$$\begin{aligned} C_t &= B_t E_t \left(\frac{\max(S_T - X, 0)}{B_T} \right) \\ &= S_t P_1 - e^{-r(T-t) - nr^o/252} X P_2, \end{aligned}$$

where

$$\begin{aligned} P_1 &= \int_X^\infty \frac{S_T}{E_t(S_T)} p_t(S_T) dS_T, \\ P_2 &= P_t(S_T > X). \end{aligned}$$

Since the probability density function is unknown under our assumptions regarding the evolution of stock and money market, Fourier inversion techniques are used to derive expressions for P_1 and P_2 (see Bakshi and Madan (2000)). For P_2 this gives,

$$P_2 = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \operatorname{Re} \left(\frac{\exp(-i\alpha \log X) \varphi(\alpha)}{i\alpha} \right) d\alpha, \quad (\text{A.1})$$

where $\varphi(\alpha)$ denotes the conditional characteristic function of the random variable $\log S_T$, i.e., $\varphi(\alpha) = E_t \exp(i\alpha \log S_T)$. The probability P_1 will be obtained later from P_2 . Given the process of $\log S_t$ above, $\varphi(\alpha)$ can be written as, with $\tau = T - t$,

$$\begin{aligned} \varphi(\alpha) &= E_t \{ \exp(i\alpha \log S_T) \}, \\ &= E_t \left\{ \exp \left(i\alpha \left[\log S_t + r\tau - \frac{1}{2} \int_t^T \sigma_u^2 du + \int_t^T \sigma_u dW_u^S + \right. \right. \right. \\ &\quad \left. \left. \left. + \sum_{i=N_t+1}^{N_T} \log Y_i - (A_T - A_t) + \sum_{i=\lfloor 252t \rfloor + 1}^{\lfloor 252T \rfloor} \log V_i \right] \right) \right\}, \\ &= E_t \left\{ \exp \left(i\alpha \left[\log S_t + r\tau - \frac{1}{2} \int_t^T \sigma_u^2 du + \int_t^T \sigma_u dW_u^S \right] \right) \right\} \times \\ &\quad E_t \left\{ \exp \left(i\alpha \left[\sum_{i=N_t+1}^{N_T} \log Y_i - (A_T - A_t) \right] \right) \right\} E_t \left\{ \exp \left(i\alpha \sum_{i=\lfloor 252t \rfloor + 1}^{\lfloor 252T \rfloor} \log V_i \right) \right\}. \end{aligned}$$

The characteristic functions of the various parts will be derived separately. The first part is equal to formula (17) in Heston (1993), i.e.,

$$\begin{aligned} &E_t \left\{ \exp \left(i\alpha \left(\log S_t + r\tau - \frac{1}{2} \int_t^T \sigma_u^2 du + \int_t^T \sigma_u dW_u^S \right) \right) \right\} \\ &= \exp \left(C(\tau; \alpha) + D(\tau; \alpha) \sigma_t^2 + i\alpha \log S_t \right), \end{aligned}$$

where

$$\begin{aligned} C(\tau; \alpha) &= r i \alpha \tau + \frac{\kappa \sigma^2}{\sigma_\sigma^2} \left\{ (\kappa - \rho \sigma_\sigma i \alpha + d) \tau - 2 \log \left(\frac{1 - g e^{d\tau}}{1 - g} \right) \right\}, \\ D(\tau; \alpha) &= \frac{\kappa - \rho \sigma_\sigma i \alpha + d}{\sigma_\sigma^2} \frac{1 - e^{d\tau}}{1 - g e^{d\tau}}, \end{aligned}$$

and,

$$\begin{aligned} g &= \frac{\kappa - \rho\sigma_\sigma i\alpha + d}{\kappa - \rho\sigma_\sigma i\alpha - d}, \\ d &= \sqrt{(\rho\sigma_\sigma i\alpha - \kappa)^2 + \sigma_\sigma^2 (i\alpha + \alpha^2)}. \end{aligned}$$

The random jump part of the model is described by means of a compensated compound Poisson process. The lognormal distribution of the jump sizes Y_i determines the characteristic function as, still with $\tau = T - t$,

$$\begin{aligned} E_t \left\{ \exp \left(i\alpha \left[\sum_{i=N_t+1}^{N_T} \log Y_i - (A_T - A_t) \right] \right) \right\} &= \\ = \exp \left\{ (A_T - A_t) / \mu_{RJ} \left[(1 + \mu_{RJ})^{i\alpha} \exp \left(\left(\frac{i\alpha}{2} \right) (i\alpha - 1) \sigma_{RJ}^2 \right) - 1 \right] - i\alpha (A_T - A_t) \right\}, \end{aligned}$$

where the compensator is given by

$$A_t = \lambda \mu_{RJ} [(1 - c)t + c \lfloor 252t \rfloor / 252].$$

Note that for integer values of 252τ , this expression does not depend on c .

The expression for the characteristic function of the fixed jump part is more tractable since (relative to the random jump part) one source of randomness disappears. The characteristic function then can be calculated, using the lognormal jump sizes V_i , as

$$E_t \left\{ \exp \left(i\alpha \sum_{i=\lfloor 252t \rfloor + 1}^{\lfloor 252T \rfloor} \log V_i \right) \right\} = \exp \left(i\alpha n r^o / 252 - \frac{1}{2} \alpha (\alpha + i) n \sigma_{OJ}^2 / 252 \right).$$

where $n = \lfloor 252T \rfloor - \lfloor 252t \rfloor$. The characteristic function of the terminal stock price is determined and can be used to obtain P_2 in the option pricing formula.

In order to obtain P_1 observe the following lemma with $Y = \log S_T$.

Lemma A.1 *Let Y be a random variable whose distribution has density p and characteristic function φ and for which $E\{\exp(Y)\} < \infty$. Define the distribution F by its survival function*

$$1 - F(z) = \int_z^\infty \frac{\exp(y)}{E\{\exp(Y)\}} p(y) dy.$$

Then, F has characteristic function $\tilde{\varphi}$ with,

$$\tilde{\varphi}(\alpha) = \frac{\varphi(\alpha - i)}{E\{\exp(Y)\}}.$$

Proof. Let Z have distribution function F and density

$$f(z) = \frac{\exp(z) p(z)}{E\{\exp(Y)\}}.$$

Now

$$\begin{aligned} \tilde{\varphi}(\alpha) &= E \exp(i\alpha Z) = \int_{-\infty}^{\infty} \exp(i\alpha z) \frac{\exp(z) p(z)}{E\{\exp(Y)\}} dz \\ &= \int_{-\infty}^{\infty} \frac{\exp(i(\alpha - i)z)}{E\{\exp(Y)\}} p(z) dz = \frac{E \exp\{i(\alpha - i)Y\}}{E\{\exp(Y)\}} \\ &= \frac{\varphi(\alpha - i)}{E\{\exp(Y)\}}, \end{aligned}$$

which concludes the proof of the Lemma.

Comparable to (A.1), this leads to,

$$P_1 = \frac{1}{2} + \frac{1}{\pi} \int_0^{\infty} \operatorname{Re} \left(\frac{\exp(-i\alpha \log X) \varphi(\alpha - i)}{i\alpha \varphi(-i)} \right) d\alpha. \quad (\text{A.2})$$